PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1915 **215**, 161-176
doi: 10.1098/rsta.1915.0006

VI. *The Lunar Diurnal Magnetic Variation, and its Change with Lunar Distance.*

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Communicated by the Astronomer Royal, F.R.S.

Received January 27,—Read February 18, 1915.

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Introduction.

In his illuminating article on Magnetism in the 'Encyclopædia Britannica' (9th ed., 1882), Balfour Stewart discussed the origin and mechanism of the short-period magnetic variations, concluding that the only tenable hypothesis was that which attributed them to currents flowing in the upper atmosphere, under the impulse of electromotive forces caused by the motion of the conducting atmosphere across the permanent terrestrial magnetic field. At several stages in the discussion use was made of the phenomena of the lunar diurnal magnetic variation, as disclosed by Broun's fine study of the subject;* to these phenomena Balfour Stewart evidently attributed considerable theoretical importance, and an origin similar as regards situation to that of the solar diurnal variations.

In 1889 Schuster† proved, by the Gaussian potential method, that the solar diurnal magnetic variations arise mainly from causes acting above the earth's surface, a demonstration which added much weight to Balfour Stewart's tentative theory. Schuster also suggested a connection between the magnetic and barometric variations, an idea which he elaborated and discussed with great cogency eighteen years later (1907).‡ The barometric changes are mainly of thermal origin, and possible differences between the character of the atmospheric motions at the earth's surface and in the upper regions may have an important bearing on the theory. In

* Broun, 'Trevandrum Magnetical Observations,' vol. 1, p. 113, 1874.

† Schuster, 'Phil. Trans.,' A, vol. 180, p. 467, 1889.

‡ Schuster, 'Phil. Trans.,' A, vol. 208, p. 163, 1907.

his later paper, SCHUSTER suggested that the lunar magnetic variations might throw light on these questions: "It is much to be desired that some systematic attempt should be made to investigate the lunar influence on the magnetic changes, for we possess at present only the vaguest information as to how the different components are affected. It is quite possible that the effects may depend on a tidal disturbance of the upper regions of the atmosphere. If so, we may expect to get a valuable test of our theory by their investigation" (*loc. cit.*, p. 181).

BALFOUR STEWART also has said that "it is impossible to refrain from associating the lunar diurnal magnetic variations either directly or indirectly with something having the type of tidal action, but in what way this influence operates we cannot tell" (*loc. cit.*, § 146).

These references to tidal action seem to have been suggested largely by a discovery of BROUN'S, that the amplitude of the lunar magnetic variation is greater at perigee than at apogee in the inverse cube ratio of the moon's distance at these epochs, "as in the theory of the tides," as he briefly concluded. The semi-diurnal character of the lunar magnetic variation agrees with this tidal hypothesis.

In 1912 VAN BEMMELEN,* in an important paper on the lunar magnetic variations, referred them (by methods similar to those used by SCHUSTER in the two papers cited) to the lunar atmospheric tide as computed from the Batavian barometric records; he showed that the electrical conductivity of the atmospheric layers in which circulate the currents responsible for the production of the lunar magnetic variations is of the same order as that calculated by SCHUSTER from the solar diurnal variations. Since the lunar barometric variation is clearly a tidal effect, the hypothesis of a tidal origin of the lunar magnetic variations is thus supported.

The most direct confirmation of this hypothesis, however, would naturally be obtained from evidence such as that adduced by BROUN, concerning the influence of lunar distance upon the amplitude of the lunar magnetic variations. BROUN'S result coincided remarkably closely with the theoretical value, but further examination indicates that this was due to a happy accident; the ratio of the moon's mean distance in the half lunations centred at perigee and apogee respectively is 1·00 : 1·07, and the inverse cube of this is 1·23; the observed values of the ratio of the mean amplitudes of the lunar magnetic variation during these epochs varied between 1·15 and 1·34, with a mean value of 1·24.

FIGEE,† making a similar investigation from the Batavian magnetic records, dissented from BROUN'S conclusion that the amplitudes vary as the tidal theory would predict.

If a result contrary to that of BROUN could be definitely established, the above theory of the lunar magnetic variations, which is so attractive in many ways, would become almost untenable. The matter being of some importance, and BROUN'S and

* VAN BEMMELEN, 'Meteorologische Zeitschrift,' vol. 5, p. 218, 1912.

† 'Batavian Magnetical and Meteorological Observations,' vol. 26, Appendix, 1903.

FIGEE'S data alone being insufficient to resolve the doubt, I have made an attempt in the present paper to decide the question, with the aid of newly computed data. The result is not so decisive as could be desired, owing to the considerable accidental errors affecting the determinations of the minute quantities concerned; but the evidence, on the whole, is confirmatory of the tidal hypothesis. It can certainly be stated that, if the amplitude of the lunar magnetic variation is inversely proportional to an integral power of the lunar distance, this power is the cube and not the square or the fourth power.

This conclusion, while important, cannot be called surprising. The investigation very clearly revealed, however, another phenomenon of a remarkable and quite unexpected nature, viz., that the phase of the lunar magnetic variation at perigee is considerably in advance of that at apogee (*i.e.*, by about 30 degrees). On referring back to BROUN'S and FIGEE'S papers, it appeared that their data confirmed this result, though BROUN, not unnaturally, made no remark on what seemed to be nothing more than an accidental feature of the observations. FIGEE, on the other hand, noticed and commented on it, but on account of variations in its amount, as determined from different portions of his material, he expressed doubt as to its reality. He appears to have gone no further with it, not even examining BROUN'S data for confirmation or otherwise. The body of evidence in its support, which is here brought forward, establishes this remarkable phenomenon beyond question.

The Change with Lunar Distance of the Amplitude of the Lunar Magnetic Variation.

If the amplitude of the lunar magnetic variation varies continuously between a maximum value at perigee and a minimum at apogee, the ratio of the mean amplitudes during two periods centred at these epochs must obviously depend on the length of the periods. The mean eccentricity of the lunar orbit being 0·055, the extreme ratio of the amplitudes (at exact perigee and apogee) on the tidal theory should be 1·39, which is the cube of $(1\cdot055 \div 0\cdot945)$: for periods of four days centred at perigee and apogee the ratio should be 1·38, while if the periods each extend over half a lunation the ratio is 1·23.

BROUN divided his material into the perigee and apogee half lunations, thus utilizing all his material, but diminishing the magnitude of the quantity to be determined. His data were ten years' hourly observations of declination at Trevandrum, in India, and he treated the summer and winter half years separately. The mean inequalities derived from the four groups of data (summer and winter, perigee and apogee) were compared according to their mean ranges, mean areas, and harmonic amplitude coefficients. The perigee-apogee ratios so obtained varied between 1·15 and 1·34, with a mean value of 1·24 from the whole material. BROUN summed up his result thus: "The ratio of the moon's mean distance from the earth in the half orbit about apogee is to that in the half orbit about perigee nearly as 1·07 to 1; as the cube of $1\cdot07 = 1\cdot23$

nearly, we see that the mean ranges of the curves, as well as the mean areas, for the two distances are in the approximate ratios of the inverse cubes of the moon's distance from the earth, as in the theory of the tides" (*loc. cit.*, § 405).

FIGEE did not determine the ratio of the mean amplitudes over half lunations, but only over periods of three days centred at apogee and perigee; hence his results should be compared with 1·39 and not with 1·23. This point seems to have been overlooked by him, for on comparing his figures with BROUN'S, he remarked that BROUN'S value of the perigee-apogee amplitude ratio (1·24) "is much smaller than that found for Batavia, 1·68, and therefore the conclusion drawn by BROUN is not allowed here 'that the mean ranges of the curves, as well as the mean areas, for the two distances, are in the approximate ratios of the inverse cubes of the moon's distance from the earth, as in the theory of the tides'" (*loc. cit.*, § 23). The figure 1·68, here referred to, is the ratio of the areas of the variation curves at perigee and apogee for declination (winter), the element chiefly affected by the moon at Batavia. The ratios for the other elements and seasons were also determined, however, though in some cases (especially that of vertical force) the whole lunar variation is so small that any effect of the kind sought for would be liable to be masked by accidental error. But in the case of horizontal force the total amplitudes, while smaller than that of declination in winter, seem to be sufficiently large to entitle the perigee-apogee ratios derived from them to some weight. The three values of this ratio which appear to be the most reliable, amongst FIGEE'S results, are consequently:

Declination (winter)	1·68.
Horizontal force (summer)	1·40.
Horizontal force (winter)	1·00.

The mean of these results* is 1·36, which is very nearly equal to the theoretical ratio 1·39; hence, we may conclude that, while it is subject to considerable accidental error, the mean result from Batavia tends, like that from Trevandrum, to the support of the tidal hypothesis as to the origin of the lunar magnetic variations.

The newly computed results, next to be described, are derived from the hourly values of the magnetic elements at five observatories, Pavlovsk, Pola, Zi-Ka-Wei, Manila, and Batavia (in each case for seven magnetically "quiet" years), treated in the manner explained in a recent paper on the lunar magnetic variations at Pavlovsk and Pola.† The amplitude ratio has been determined both from short periods of three

* The other results obtained by FIGEE are as follows: declination (summer) 1·09, vertical force (summer) 1·97, vertical force (winter) 0·89. The mean of these values is 1·32, but on account of the smaller amplitude of these variations, not much weight can be given to this mean value.

† CHAPMAN, 'Phil. Trans.,' A, 214, pp. 295–317 (1914). For the purpose of the perigee-apogee investigation the work described in that paper is useful up to the end of § 4.

The seven years dealt with are 1897–1903, except in the case of Batavia, where on account of interruptions in the observations the years 1899–1901 were replaced by the correspondingly quiet years of the previous sunspot cycle, 1888–1890.

or four days centred at apogee and perigee, and also from half lunations, as in BROUN'S investigation. In order not to waste labour on computations which were not likely to afford an accurate result, owing to the relative magnitude of the amplitude change and the accidental errors, only those elements have been dealt with (and during those seasons*) for which the range of the semi-diurnal lunar magnetic variation approximated to at least 2γ (2.10^{-5} C.G.S.).

The results as regards amplitude are given in Tables I. and II., the former applying to the shorter periods about apogee and perigee, and the latter to the half lunations. The number of days contributing to each individual value of the semi-amplitude in Table I. was approximately 100, and in Table II., approximately 400. The theoretical value of the amplitude ratio in the former case is 1·38, and in the latter, 1·23. The means of the observed values are as follows:—

			Theoretical value.
Short periods	Declination (six determinations)	1·33	1·38
	Horizontal force (five determinations)	1·43	
	Vertical force (four determinations)	1·31	
Half lunations	Declination (six determinations)	1·23	1·23
	Horizontal force (six determinations)	0·98	
	Vertical force (four determinations)	1·21	

The individual results in Tables I. and II., on which these means are based, show considerable discordances, and better data are much to be desired. The accidental errors in the determination of the minute lunar magnetic variations, especially from a comparatively small number of days, are considerable; in one or two cases, indeed, some of the computed semi-amplitudes are palpably erroneous, and have been discarded in taking the means from the tables. The discordance (including even the rather surprising discordance of the second horizontal force mean value, 0·98) must, I think, be attributed to the fortuitous effect of magnetic disturbances, and this can be eliminated only by determining the lunar variation over the average of a considerably longer period of time. It would be well if observatories would undertake such a reduction of their own observations, but meanwhile the only available data are those here communicated.

Apart, then, from the fifth mean value tabulated above,† the present results, taken in conjunction with BROUN'S and FIGEE'S data as previously discussed, may be said to confirm with reasonable probability the hypothesis of tidal action. The material is certainly not sufficient to determine the magnitude of the ratio of amplitudes at perigee and apogee to within a few per cent., as is desirable. It would

* The year was divided up into three parts, summer, winter, and equinox, comprising May–August, November–February, and the intervening months, respectively.

† It does not seem probable that the smallness of this result for horizontal force represents a real phenomenon, especially as the second of the above mean values (1·43) would tend to the opposite conclusion.

not be unreasonable to conclude from the present data that the effect of lunar distance may be a little less than the tidal theory would indicate;* but, at any rate, it seems clear that we may conclude that if the amplitude of the lunar magnetic variation is inversely proportional to the n^{th} power of the moon's distance, where n is integral, then n can have no other value than three.†

The Change with Lunar Distance of the Phase of the Lunar Magnetic Variation.

In the computations by which the lunar magnetic variation data, discussed here and in a former paper, were determined, the hourly values of the magnetic elements, after being freed from the solar diurnal variation, were written out in rows of twenty-five, the first value on each row corresponding to the civil hour nearest to the moon's meridian transit on that day.‡ Hence the origin of lunar time is, in the mean, at the hour of lunar transit and is independent of the longitude of the moon in its orbit. We shall represent the lunar magnetic variation by $\sin(2t+\theta)$, where t, measured from the origin just mentioned, increases by 2π in a lunar day: then θ is the phase of the variation, which is found to vary with the longitude of the moon. We shall denote its values at perigee and apogee by θ_P and θ_A, and its mean values during the half lunations centred at these epochs by $\overline{\theta}_P$ and $\overline{\theta}_A$. The differences $\theta_P-\theta_A$ and $\overline{\theta}_P-\overline{\theta}_A$ corresponding to the various observatories, elements, and seasons dealt with in Tables I. and II. are given in Tables III. and IV. respectively; in practically every case they are positive, indicating an acceleration of phase at perigee. As a slight guide to the probable reliability of the determined phase angles, the mean of the amplitudes given in Tables I. and II. are tabulated in Tables III. and IV. respectively.

The mean values of $\theta_P-\theta_A$ from Tables III. (*a*), (*b*), and (*c*), which, being derived from short periods, should show practically the whole phase change between these epochs, are

	Degrees.
Declination (six determinations)	+26
Horizontal force (five determinations)	+30
Vertical force (four determinations)	+18

Similarly the mean value of $\overline{\theta}_P-\overline{\theta}_A$ are, from Table IV.,

	Degrees.
Declination (six determinations)	+8·2
Horizontal force (six determinations)	+9·8
Vertical force (four determinations)	+8·9

* A possible explanation of such a conclusion, if it were substantiated, might be framed along the lines indicated on p. 168, last paragraph.

† The amplitude ratios for "short periods" if n had the values 2, 4 would be 1·23, 1·51 respectively, and for half lunations, 1·14 and 1·31 respectively, which seem to be outside the probable limits of error of the observed results.

‡ *Cf.* 'Phil. Trans.,' A, vol. 214, § 3.

These mean results may be compared with individual results from the quoted memoirs by BROUN and FIGEE. The values from BROUN's paper are

		Degrees.
Trevandrum declination	October–April	$\bar{\theta}_P - \bar{\theta}_A = 21$
	May–September	$\bar{\theta}_P - \bar{\theta}_A = 14$

These differences (on which BROUN made no remark) confirm the phase change indicated in Table IV., though the amount is larger than the value there determined.

FIGEE gives the phase angles at perigee and apogee only for declination (winter), dividing his sixteen years' data into various groups as follows :—

	Degrees.
Nine maximum sunspot years	$\theta_P - \theta_A = 42$
Seven minimum sunspot years	33
Eight odd years of the series	54
Eight even years of the series	11
Whole sixteen years	37

He remarked: "The first formulæ reveal the remarkable property that the occurrence of the maximum of the semi-diurnal wave was accelerated by the increasing magnetic force exercised by the moon from apogee to perigee, the amount of the acceleration being more than one hour, with a striking accordance in the two periods chosen (maximum and minimum). It was to be expected that a subdivision in two other series (odd and even years) should show a similar accordance, which unfortunately was not the case, as may be seen from the above figures. By this the reality of an acceleration of the occurrence of the maximum with decreasing distance of the moon from the earth is made less probable, though certainly suggested by the above figures."

Discussion.

The whole body of evidence here collected makes clear the reality of this remarkable phase change, so that the differences between FIGEE's various results would seem to be only accidental, large though some of them are. The magnitude of the phase change is not very exactly determined, but it appears to be approximately 30 degrees between perigee and apogee. The value of $(\bar{\theta}_P - \bar{\theta}_A)/(\theta_P - \theta_A)$, if accurately known, should afford information as to how the phase varies between these epochs. If the phase angle is harmonically periodic during a lunation, so that the formula for the lunar variation is

$$\sin\left(2t + c\cos\overline{pt+\alpha} + \beta\right)$$

(where $1/p$ is the number of days in a lunation), then the mean phase angle during a half lunation should be $2/\pi$ times the phase angle at the middle point of this period, at whatever epoch this middle point may be. Hence the value of $(\bar{\theta}_P - \bar{\theta}_A)/(\theta_P - \theta_A)$ should, on this hypothesis, be $2/\pi$ also; in the present case, if the mean results from Tables III. and IV. be taken, its value is *less* than $2/\pi$—BROUN'S value of $\bar{\theta}_P - \bar{\theta}_A$ would satisfy the relation more exactly. Without much more accurate data, however, it would be very unwise to conclude definitely that the phase angle does not vary harmonically throughout the lunation.

According to the theory of the lunar magnetic variations which was outlined in the Introduction to this paper, they are primarily due to a lunar atmospheric tide, which produces the electric currents responsible for the magnetic variations. It is a question of ascertainable fact whether or not the changes in phase angle (with change of lunar distance) which are found in the lunar magnetic variations are already present in the lunar atmospheric tide, as revealed by the barometric records. The computations necessary to determine this point have not, however, yet been made, and therefore in the present discussion it is advisable to consider what information in the matter may be derived from general theory.

In the case of the atmospheric tides, the tidal theory appropriate to a uniform ocean is presumably applicable. A change of phase in the tide appears possible only if frictional forces are acting, and these would produce a retardation of phase, the magnitude of which depends only on the period and not on the amplitude of the tidal oscillation. In the case of the lunar atmospheric tide it is chiefly the amplitude which alters (over a total range of 40 per cent.), while the period changes only by about $\frac{1}{3}$ per cent., being shorter at perigee than at apogee; this should result in a retardation of phase at perigee, though of negligible amount.

The matter may be regarded otherwise, as follows: the main lunar semi-diurnal tides are analysed into M_2, an invariable semi-diurnal tide $A \sin(2t-\alpha)$, and N the principal lunar elliptic tide (see DARWIN'S 'Collected Papers,' vol. 1, p. 20), the amplitude of which is approximately one-fifth of M, and which may be described as semi-diurnal, but with a slowly changing phase angle which increases through 2π during each lunation—it may be written $\frac{1}{5}A \sin(2t+pt+\beta)$. For convenience we may suppose the origin of t chosen so that $\alpha = 0$. Theory does not definitely predict what, in the actual case, will be the value of β, the difference in phase between M_2 and N at (say) perigee, but it would be expected to be quite small. The combination of M_2 and N results in a semi-diurnal oscillation, the amplitude of which changes through a total range of 40 per cent., while the phase varies through approximately 23 degrees, $11\frac{1}{2}$ degrees on either side of the mean. The maximum and minimum amplitudes occur at the epochs of mean phase. If at perigee $\beta = 0$, the amplitude change would coincide with that observed in the lunar magnetic variation, but there would be no corresponding phase variation; if $\beta = \frac{1}{2}\pi$, there would be no change of amplitude between perigee and apogee, but there would be a phase change of 23 degrees between apogee and perigee, in the direction observed in the lunar magnetic variation. For intermediate values of β intermediate states would prevail—*e.g.*, if $\beta = \frac{1}{4}\pi$, the amplitude change is reduced to a ratio 1·30, and there is a phase change of 17 degrees between perigee and apogee. The observed amplitude change in the lunar magnetic variation might be a little less than 1·39, though hardly so much below this as 1·30; this would be compatible, on the above view, with a change of phase of something less than 17 degrees. Possibly in this way a part of the observed phase change in the lunar magnetic variation might be explained (a more accurate determination of the amplitude change would decide this point more definitely), but in no circumstances could the whole of it be thus accounted for. Probably the explanation which must be sought elsewhere will account for the whole of the phenomenon under discussion.

Before leaving the consideration of the atmospheric tide, its actually observed phase, as determined from forty years' barometric observations at Batavia, may be adverted to. The lunar diurnal variation of barometric pressure was found to be

$$0{\cdot}0628 \cos(2t+65^\circ),$$

t being reckoned from the local time of the moon's transit. The fact that the tide is in advance of the moon is difficult to understand, and it is conceivable that the unknown cause which thus accelerates the tide also has some connection with the perigee-apogee phase change in the lunar magnetic variations.

So far no real light on the origin of this phase change has been found, since theoretical reasoning indicates no such change in the atmospheric tide which is supposed to produce the lunar magnetic variations. Whether the phase of the tide does or does not so change is, as already stated, unknown; if it is found to vary correspondingly with the magnetic variations, the tidal theory of the latter would be strengthened, though the phase change in the tides themselves would offer a problem demanding solution. In the present state of ignorance, however, it is natural to consider whether, if the phase of the atmospheric tide is independent of lunar distance, the phase change in the magnetic variations could be accounted for in any electromagnetic way. Self-induction is the only possible cause which suggests itself, and this, unfortunately, seems as incapable of offering an explanation as tidal friction was found to be; the effect of self-induction is to produce a phase retardation which is independent of the amplitude of a periodic variation, but which diminishes with the period. In the present case the variation of period would not account in this way for more than 1 degree change of phase.

What has been said as to the failure of tidal friction to explain the phase change in the lunar magnetic variation applies also to tides in the substance of the earth, so that even if (as VAN BEMMELEN supposed in his memoir already cited) such body tides have a part in the production of the lunar magnetic variation, they do not aid in the explanation of the phase change under discussion. VAN BEMMELEN in an amending paper ('Met. Zeitschr.,' 12, p. 589, 1913) withdrew his earlier suggestion of a primary internal magnetic field concerned in the lunar variations; it may be remarked incidentally that such a field is rendered unlikely by the magnitude of the components other than semi-diurnal in the magnetic variations.* These appear to be excited by the semi-diurnal atmospheric tide in conjunction with a variable electrical conductivity of the atmosphere, depending on the solar hour angle; their phases change by multiples of 2π in the course of a lunation, according to ascertained laws. While only secondary phenomena, they are comparable in magnitude with the main (semi-diurnal) component of the lunar variation, so much so, in fact, that it is hardly possible for more than a very small fraction of the latter to be due to internal causes, since this portion would not account for any part of the secondary components.

Since perigee and apogee occur at all phases of the moon during a sufficiently long period of time, no explanation of the perigee-apogee phase change in the magnetic variations can be looked for in any direct solar action.

Thus far, therefore, the attempt to assign a known cause to this remarkable phenomena has been unsuccessful. It remains to make one more suggestion, which is at once very tentative and far from definite. We should naturally suppose that the tidal effect of the moon is to produce a lunar magnetic variation $A \cos(2t+\alpha)$, the amplitude (A) of which undergoes a regular variation of the kind observed, but which shows no change of phase; the observed phase change might be produced if in some other way the moon produced a semi-diurnal magnetic variation of type $B \cos(2t+\beta)$, where β exceeds α by about 90 degrees, provided that B/A is fairly small and that B increases more quickly than A as the moon's distance diminishes. The combination would slightly increase the theoretical variation of amplitude, but this need only be by a small amount.

Not much importance can be attached to this suggestion, however, since it is very difficult to conceive of any way, other than tidal, in which the moon could produce a semi-diurnal variation of terrestrial magnetism. Certainly no appreciable direct magnetic effect of the moon is at all likely, nor would it, in any case, supply the desired variation in the present instance, since its effect would be diurnal and not semi-diurnal. The matter must therefore remain a mystery for the present, but it is a mystery whose solution may be the clue to some important new fact relating to magnetism of the atmospheric tides; it is with this hope that I venture to publish the preceding very inconclusive discussion.

In conclusion it is a pleasure to acknowledge the assistance which has been placed at my disposal, in the execution of the computations involved in this paper, by the Government Grant Committee of the Royal Society.

* *Cf.* 'Phil. Trans.,' A, vol. 213, p. 279, 1913.

TABLE I.—Ratio of Mean Semi-amplitudes of the Lunar Semi-diurnal Variation of Terrestrial Magnetism, during a number of periods of three or four days centred at Perigee and Apogee respectively.

(*a*) Declination West.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Pavlovsk	Summer	238	159	1·50
Pola	Summer	225	172	1·31
Zi-Ka-Wei	Summer	293	200	1·46
	Equinox	145	170	0·85
Manila	Summer	191	120	1·59
Batavia	Winter	291	245	1·19
	Equinox	130	(54)	(2·4)
			Mean	1·33 ± 0·08

(*b*) Horizontal Force.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Pavlovsk	Summer	137	72	1·91
Pola	Summer	172	128	1·35
	Equinox	166	100	1·65
Zi-Ka-Wei	Winter	(93)	175	(0·53)
Manila	Winter	171	128	1·33
Batavia	Winter	118	131	0·90
			Mean	1·43 ± 0·12

(*c*) Vertical Force.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Zi-Ka-Wei	Summer	152	101	1·51
	Equinox	142	135	1·05
Manila	Winter	99	99	1·00
	Equinox	122	72	1·69
			Mean	1·31 ± 0·14

TABLE II.—Ratio of Mean Semi-amplitudes of the Lunar Semi-diurnal Variation of Terrestrial Magnetism, during a number of half months centred at Perigee and Apogee respectively.

(*a*) Declination West.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Pavlovsk	Summer	147	117	1·26
Pola	Summer	168	138	1·21
Zi-Ka-Wei . . {	Summer	266	198	1·34
	Equinox	164	159	1·03
Manila	Summer	161	117	1·38
Batavia . . . {	Winter	282	245	1·15
	Equinox	111	(24)	(4·63)
			Mean . . .	1·23 ± 0·04

(*b*) Horizontal Force.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Pavlovsk	Summer	108	92	1·18
Pola . . . . . {	Summer	144	149	0·97
	Equinox	88	88	1·00
Zi-Ka-Wei . . .	Winter	126	147	0·86
Manila	Winter	130	142	0·92
Batavia	Winter	139	145	0·96
			Mean . . .	0·98 ± 0·03

(*c*) Vertical Force.

Observatory.	Season.	Semi-amplitude in force units, 10^{-7} C.G.S.		Ratio.
		Perigee.	Apogee.	
Zi-Ka-Wei . . . {	Summer	132	92	1·44
	Equinox	132	116	1·14
Manila . . . . {	Winter	101	100	1·01
	Equinox	117	93	1·26
			Mean . . .	1·21 ± 0·07

TABLE III.—Differences in Phase Angle θ in the Formula $\sin(2t+\theta)$ for the Lunar Semi-diurnal Magnetic Variation at Perigee and Apogee, as determined from a number of short periods centred at these epochs.

(*a*) Declination West.

Observatory.	Season.	Mean amplitude (from Table I. (*a*)) in force units, 10^{-7} C.G.S.	Phase difference, $\theta_P - \theta_A$.
Pavlovsk	Summer	198	21°
Pola	Summer	198	36
Zi-Ka-Wei	Summer	246	12
	Equinox	158	43
Manila	Summer	156	26
Batavia	Winter	268	22
	Equinox	(92)	(15)
		Mean	$+26° \pm 3°\cdot4$

(*b*) Horizontal Force.

Observatory.	Season.	Mean amplitude (from Table I. (*b*)) in force units, 10^{-7} C.G.S.	Phase difference, $\theta_P - \theta_A$.
Pavlovsk	Summer	104	+31°
Pola	Summer	150	− 2
	Equinox	133	+56
Zi-Ka-Wei	Winter	(134)	(27)
Manila	Winter	150	28
Batavia	Winter	125	38
		Mean	$+30° \pm 6°\cdot0$

(*c*) Vertical Force.

Observatory.	Season.	Mean amplitude (from Table I. (*c*)) in force units, 10^{-7} C.G.S.	Phase difference, $\theta_P - \theta_A$.
Zi-Ka-Wei	Summer	126	24°·1
	Equinox	138	27·7
Manila	Winter	99	13·0
	Equinox	97	7·9
		Mean	$18°\cdot2 \pm 3°\cdot8$

TABLE IV.—Differences in the Phase Angle θ in the Formula $\sin(2t+\theta)$ for the Lunar Semi-diurnal Magnetic Variation, at Perigee and Apogee, as determined from a number of half-lunations centred at these epochs.

(*a*) Declination West.

Observatory.	Season.	Mean amplitude (from Table II. (*a*)) in force units, 10^{-7} C.G.S.	Phase difference, $\theta_P - \theta_A$.
Pavlovsk	Summer	132	8°
Pola	Summer	153	6
Zi-Ka-Wei	Summer	232	6
	Equinox	162	17
Manila	Summer	139	5
Batavia	Winter	264	8
		Mean	$+8^\circ \cdot 2 \pm 1^\circ \cdot 0$

(*b*) Horizontal Force.

Observatory.	Season.	Mean amplitude (from Table II. (*b*)) in force units, 10^{-7} C.G.S.	Phase difference, $\theta_P - \theta_A$.
Pavlovsk	Summer	100	2°
Pola	Summer	146	-10
	Equinox	88	28
Zi-Ka-Wei	Winter	136	18
Manila	Winter	136	10
Batavia	Winter	142	11
		Mean	$+9^\circ \cdot 8 \pm 3^\circ \cdot 5$

(*c*) Vertical Force.

Observatory.	Season.	Mean amplitude (from Table II. (*c*)) in force units, 10^{-7} C.G.S.	Phase difference $\theta_P - \theta_A$.
Zi-Ka-Wei	Summer	112	$12^\circ \cdot 2$
	Equinox	124	$7 \cdot 2$
Manila	Winter	100	$9 \cdot 3$
	Equinox	105	$6 \cdot 9$
		Mean	$8^\circ \cdot 9 \pm 0^\circ \cdot 9$

NOTE ADDED MARCH, 1915.

(1) *The Seasonal Changes of* $\theta_P - \theta_A$.

If the values of $\theta_P - \theta_A$ from Tables III. (*a*), (*b*) and IV. (*a*), (*b*) be grouped together according to their season, it will be found that the mean value at the equinoxes is about double that at the other seasons, the difference being quite noticeable; this was kindly pointed out to me by Prof. H. H. TURNER, F.R.S. In order to examine this point a little further (since the result mentioned rests on material which is much more scanty for the equinox than for the other seasons), I caused the results given in Tables I. to IV. (*c*) (for vertical force), to be computed. These were not in the paper as originally communicated to the Royal Society, as I did not then know that the amplitudes of the variations for this element and these seasons reached the value 2γ.

If now all the values of $\theta_P - \theta_A$ in Tables III. and IV. are grouped together according to season, and the means taken, we obtain the following results:—

Mean Values of $\theta_P - \theta_A$ *from Table III.* (*short periods*).

	Degrees.			
Summer	$21 \pm 3 \cdot 2$	from	seven	determinations.
Winter	$26 \pm 2 \cdot 7$	,,	five	,,
Equinox	$30 \pm 6 \cdot 6$	,,	five	,,

Mean Values of $\theta_P - \theta_A$ *from Table IV.* (*long periods*).

	Degrees.			
Summer*	$6 \cdot 5 \pm 0 \cdot 9$	from	six	determinations.
Winter	$11 \cdot 2 \pm 1 \cdot 1$	,,	five	,,
Equinox	$14 \cdot 7 \pm 2 \cdot 6$	,,	five	,,

Both groups of mean values show a progression in the phase change from summer to winter, with maxima at the equinoxes. If this is really the case, the phenomenon should make success in the search for the cause of the phase change much more probable; at present, however, the magnitude of the probable errors are such as to cast doubt on the reality of the seasonal change, and, until it is more definitely established, any attempt at its explanation would be premature.

(2) *The Lunar Atmospheric Tide.*

Further information concerning this is contained in a memoir by WAGNER,† who has discussed the barometric observations for the years 1903–8 made at Samoa. The mean result for the semi-diurnal tide is, in millimetres,

$$0 \cdot 039 \sin (2t + 33 \text{ degrees}).$$

* Omitting the negative value -10 for Pola horizontal force.

† G. WAGNER, 'Göttingen Abh.,' IX., 4 (1913).

The seasonal and synodic changes in this variation are discussed, and also the change during the anomalistic month (from apogee to perigee). But the result in the latter case is (in the words of the author) disappointing, as no clear progression is made out, presumably owing to the somewhat small amount of material used for discussion. Such apparent change as the data show is a *retardation* from apogee to perigee, but this cannot be relied on. This first attempt to settle the point mentioned on p. 168 is therefore inconclusive, and further work on it is desirable.

The effect of lunar declination on the tide is also dealt with by WAGNER, without any decided result. Since a tidal effect may be regarded as due to a moon and anti-moon, one of which will be in Northerly declination when the other is Southerly, and *vice versâ*, it does not seem probable that any explanation of the perigee-apogee magnetic phase change is to be found in the variation of lunar declination.